

ELEMENTARY LINEAR ALGEBRA – SET 2

Polynomials, rational functions, partial fractions

1. Find all integer roots of the following real polynomials:

$$(a) x^3 + 3x^2 - 4, \quad (b) x^4 - 2x^3 - 8x - 12, \quad (c) x^4 - x^2 - 2$$

2. Find all rational roots of the following real polynomials:

$$(a) 4x^4 + 4x^3 + 3x^2 - x - 1, \quad (b) x^4 - 5x^2 + 4, \quad (c) 6x^4 + 7x^2 + 2$$

3. Find all roots of the following real polynomials:

$$(a) x^4 + x^3 - 3x^2 - 4x - 4, \quad (b) x^4 + x^3 + x^2 + 3x - 6$$

4. Find all roots of the following complex polynomials, knowing one of their roots:

$$(a) z^4 - 4z^3 + 12z^2 - 16z + 15, \quad z_1 = 1 + 2i \quad (b) z^4 - 4z^3 + 16z^2 - 12z + 39, \quad z_1 = i\sqrt{3}$$

5. For given real polynomials P, Q find the remainder in the division of P by Q without using the polynomial long division:

$$P(x) = x^8 + 3x^5 + x^2 + 4, \quad Q(x) = x^2 - 1$$

6. Factor the following real polynomials into irreducible real factors:

$$(a) x^3 + x^2 + x + 1, \quad (b) x^4 - 4x^3 + 8x, \quad (c) x^4 + 5x^2 + 6$$

7. Factor the following complex polynomials into irreducible complex factors:

$$(a) z^3 + z^2 + z + 1, \quad (b) x^4 + 5x^2 + 6, \quad (c) x^6 + 8$$

8. Decompose the following real rational functions into real partial fractions:

$$(a) \frac{x}{(x^2 + 1)(x - 2)}, \quad (b) \frac{x - 1}{x^3 - 1}, \quad (c) \frac{1}{(x^2 + 1)(x - 1)(x + 2)}$$

9. Decompose the following complex rational functions into complex partial fractions:

$$(a) \frac{1}{z^3 - 2z^2 + z - 2}, \quad (b) \frac{z^2}{z^3 - 1}, \quad (c) \frac{1}{(z^2 + 1)(z - 1)}$$

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